MODULE -1

1. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ show that $xu_x + yu_y = 1$
2. if $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$
3. if $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
4. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$
5. Find the total derivative of the following function and also verify the result by direct substitution method $z = xy^2 + x^2y$ where $x = at$, $y = 2at$
6. if $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
7. if $u = f(x-y, y-z, z-x)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
8. if $z = f(x,y)$ where $x = rcos\theta\ and\ y = rsin\theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$
9. If $u = f(\frac{y-x}{xy}, \frac{z-x}{xz})$ Prove that $x^2u_x + y^2u_y + z^2u_z = 0$
10. If $u = f(2x-3y, 3y-4z, 4z-2x)$, Show that $6u_x + 4u_y + 3u_z = 0$
11. If $z = f(x,y)$, $x = u - v, y = uv$ show that $(u+v)\frac{\partial z}{\partial x} = u\frac{\partial z}{\partial u} - v\frac{\partial z}{\partial v}$
12. If $u = xy + yz + zx$ where $x = cost, y = sint, z = t$ find $\frac{\partial u}{\partial t}$ at $t = \frac{\pi}{4}$
13. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$
14. if $x = rsin\theta\ cos\phi$, $y = sin\theta\ \ sin\phi$, $z = rcos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 sin\theta$
15. if $x + y + z = u$, $y + z = v\ and\ z = uvw$, find the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$
16. if $x + y + z = u$, $y + z = uv\ and\ z = uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$
17. if $u = \frac{yz}{x}, v = \frac{zx}{y}, w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$
18. if $u = x + 3y^2 - z^3$, $v = 4x^2yz$, $w = 2z^2 - xy$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$
19. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$
20. Find the maximum and minimum values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$
21. Examine the function $xy(a-x-y)$ for extreme values
22. Examine the function $sin + siny + \sin(x+y)$ for extreme values
23. Expand $e^x\ \log(1+y)$ in the powers of x and y upto terms of three degree
24. Expand $x^2y + 3y - 2$ in the powers of $(x-1)$ and $(y+2)$ using Taylor's theorem
25. Expand the following function as for as terms of three degree
    i. $sinxcosy$ ii. $sinxsin\ y$ iii. $e^x siny$
26. Expand $e^x cosy$ in a Taylor's series about the point $(1, \frac{\pi}{4})$

MODULE-2

27. Find the directional derivative of the following
    i. $\phi = x^2yz + 4xz^2$ at $(1,-2,-1)$ along $2i - j - 2k$
    ii. $\phi = 4xz^3 - 3x^2y^2z$ at $(2,-1,2)$ along $2i - 3j + 6k$
28. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ along $(2,-1,2)$
29. Find the values of constants a and b such that the surfaces $ax^2 - byz = (a+2)x$ and $4x^2y + z^3 = 4$ are orthogonal at a point $(1,-1,2)$
30. Find $div\vec{F}$ and $curl\vec{F}$ where $\vec{F} = \nabla(x^3 + y^3 + Z^3 - 3xyz)$
31. If $\vec{F} = \nabla(xy^3z^2)$ find $div\vec{F}$ and $curl\vec{F}$ at the point $(1,-1,1)$
32. If $\vec{F} = (\mathrm{x} + \mathrm{y} + 1)i + j - (x+y)k$, show that $\vec{F}.curl\vec{F} = 0$
33. Show that $\vec{F} = (\mathrm{y} + \mathrm{z})i + (z+x)j + (x+y)k$ is irrotational . Also Find a scalar function ϕ such that $\vec{F} = \nabla\phi$
34. Show that $\vec{F} = (2\mathrm{xy}^2 + \mathrm{yz})i + (2x^2y + xz + 2yz^2)j + (2y^2z + xy)k$ is conservative force field. Find scalar potential
35. Find the value of constant a such that the vector field, $\vec{F} = (\mathrm{axy} - \mathrm{z}^3)i + (a-2)x^2j(1-a)xz^2k$ is irrotational and hence Find a scalar function ϕ such that $\vec{F} = \nabla\phi$
36. Find the constants a and b such that $\vec{F} = (\mathrm{axy} + \mathrm{z}^3)i + (3x^2 - z)j + (bxz^2 - y)k$ is irrotational .Also find a scalar function ϕ such that $\vec{F} = \nabla\phi$
37. If $\vec{F} = (\mathrm{x} + \mathrm{y} + \mathrm{az})i + (bx + 2y - z)j + (x + cy + 2z)k$ find the a, b, c such that $curl\vec{F} = 0$ and then find ϕ such that $\vec{F} = \nabla\phi$
38. Prove that Spherical co-ordinate system is orthogonal
39. Prove that cylindrical co-ordinate system is orthogonal
40. Express the vector field $2yI - zJ - 3xK$ spherical polar co-ordinate system
41. Express the vector field $A = zi - 2xj + yk$ in cylindrical polar co-ordinate system
42. Express the vector field $A = 2xi - 3y^2j + xzk$ in cylindrical polar co-ordinate system
43. Express the vector field $F = yi - zj + xk$ spherical polar co-ordinate system and hence find F_r, F_θ, F_ϕ

MODULE -3

44. Test the consistency and the find the solution.

I. $x+y+z=9$; $x-2y+2z=8$; $2x+y-z=3$

II. $2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$

III. $x_1+x_2+x_3+x_4=2$; $2x_1-x_2+2x_3-x_4=-5$;

$3x_1+2x_2+3x_3+4x_4=7$; $x_1-2x_2-3x_3+2x_4=5$

IV. $x-y+z=2$; $3x-y+2z=-6$; $3x+y+z=-18$

V. $x+y+z=-3$; $3x+y-2z=-2$; $2x+4y+7z=7$

45. Investigate the values of λ and μ so that following system of equation possesses (I) Unique (II) Infinitely many solution (III) No solution

I. $2x+3y+5z=9$; $7x+3y-2z=8$; $2x+3y+\lambda z=\mu$

II. $x+y+z=6$; $x+2y+3z=8$; $x+2y+\lambda z=\mu$

III. $x+2y+3z=6$; $x+3y+5z=9$; $2x+5y+\lambda z=\mu$

46. By using Gauss elimination method solve

$2x+20y-2z+44=0$; $10x+2y-z-9=0$; $-2x+3y+10z-22=0$

47. Using Gauss Jordan method to solve.

I. $x+y+z=9$; $x-2y+3z=8$; $2x+y-z=3$

II. $2x+y+4z=12$; $4x+11y-z=33$; $8x-3y+3z=20$

III. $2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$

VI. $3x+4y+5z=3$; $2x-y+8z=13$; $5x-2y+7z=20$

VII. $2x+5y+7z=52$; $2x+y-z=0$; $x+y+z=9$

48. Find the rank of the following matrices.

a) $\begin{bmatrix} 2 & -1 & 1 \\ 3 & -1 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -1 & 3 & -1 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 1 & 0 & 2 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 5 & -4 & 6 \\ 1 & 2 & -2 & 2 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ f) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & 5 & -4 & 7 \\ -1 & -2 & -1 & 2 \\ 3 & 3 & -5 & 10 \end{bmatrix}$

g) $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ h) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ i) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$

j) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ k) $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 1 & 2 & 3 & 11 \end{bmatrix}$ l) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 4 \\ 2 & 1 & 5 \end{bmatrix}$

m) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ n) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ o) $\begin{bmatrix} 3 & 2 & 1 & 4 \\ 1 & 4 & 3 & 2 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$

p) $\begin{bmatrix} 8 & 2 & 1 & 6 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 3 \\ 5 & 1 & 1 & 4 \end{bmatrix}$ q) $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ 4 & -1 & -3 & 2 \end{bmatrix}$

ANSWERS: (48)

a) 3	b) 4	c) 4	d) 2	e) 3	f) 3
g) 3	h) 2	i) 3	j) 2	k) 3	l) 2
m) 2	n) 3	o) 2	p) 3	q) 3	

49. Find the constant b if the rank of $\begin{bmatrix} 1 & 1 & -1 & 0 \\ 4 & 4 & -3 & 1 \\ b & 2 & 2 & 2 \\ 9 & 9 & b & 3 \end{bmatrix}$ is 3.

50. Find all Eigen value and corresponding Eigen vector for the following matrices

(i) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

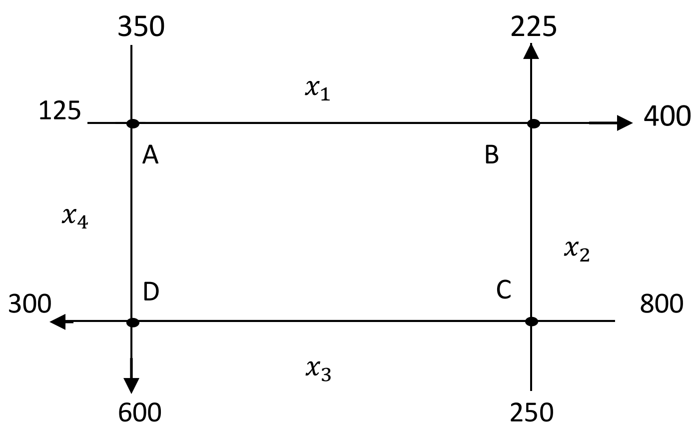
(III) $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

51. Diagonalize the following Matrices:

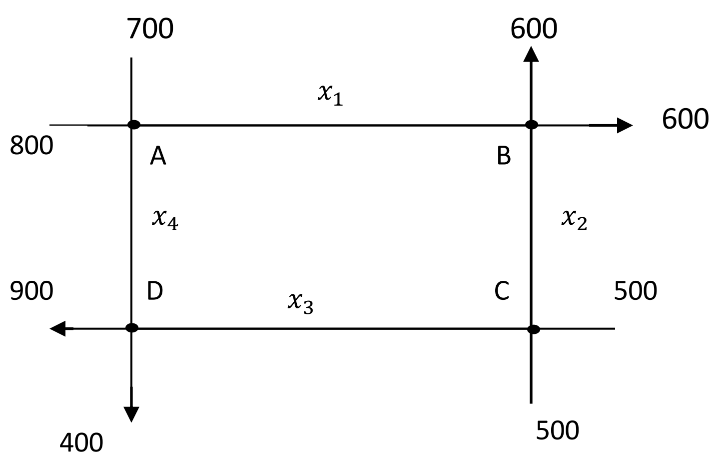
(i) $\begin{bmatrix} -19 & 17 \\ -42 & 16 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$

52. Find the Model matrix of $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and verify its diagonalization.

53. The flow of traffic through a network of streets is shown below:
(i) Find x_1, x_2, x_3, x_4 to balance the traffic flow.
(ii) Find the traffic flow at $x_4 = 0$
(iii) Find the traffic flow at $x_4 = 100$

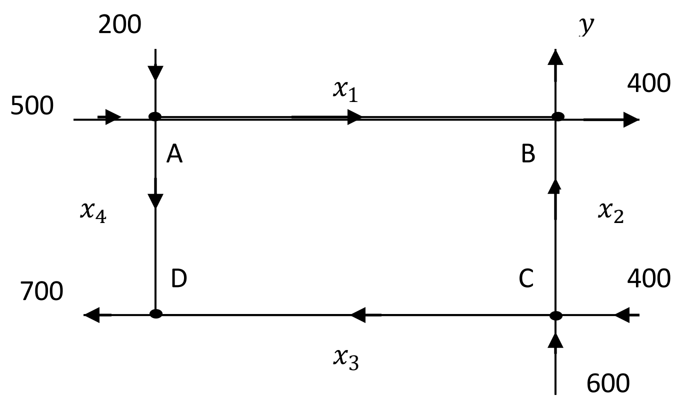


54. Balance the traffic flow at each intersection.

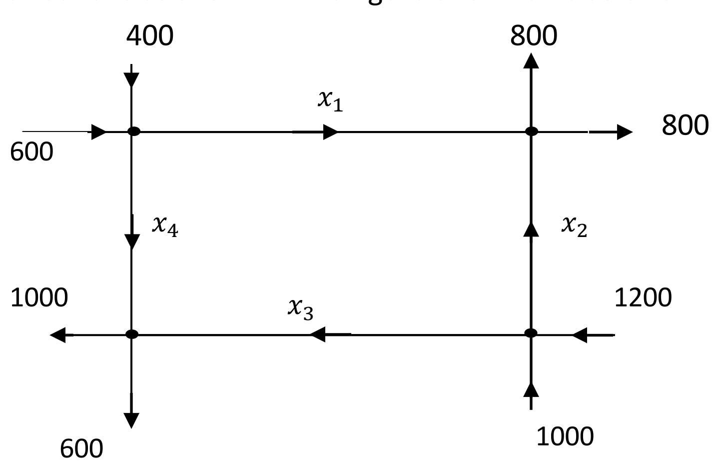


55. (i) What flow to the north should the traffic light at the intersection B let through to balance the traffic flow in this network?

(i) Assuming that the traffic light at B has been set to balance the total flow in and flow out, what should the flow be through the inner branches?



56. Write the system of linear equations of the traffic flow in the net of one-way street directions as shown in the figure and find its solution.



Module -4

57. Show that w is a subspace of $V = R^3$, where w consists of those vectors each whose sum of components is zero, i.e. $w = \{(a, b, c): a + b + c = 0\}$
58. Show that $w = \{(a, b, c): a \geq 0\}$ is not a subspace of $V = R^3$
59. Prove that the subset $w = \{(x, y, z): x - 3y + 4z = 0$ of the vector space R^3 is subspace of R^3
60. Let $V = R^3$ be a vector space and consider the subset w of V consisting of the vector of the form (a, a^2, b) , where the second component is the square of the first. is w a subspace of V
61. Let $V = R^3$ the Euclidean 3- space. let $w = \{(x, y, z): ax + by + cz = 0; x, y, z \in R\}$, a, b, c being real numbers show that w is a subspace of V
62. Write the vector $v = (1, -2, 5)$ as a linear combinations of the vectors
$$e_1 = (1,1,1), \quad e_2 = (1,2,3), \quad e_3 = (2, -1,1)$$
63. Write the vector $v = (2, -5, 3)$ as a linear combinations of the vectors
$$e_1 = (1, -3, 2), \quad e_2 = (2, -4, -1), \quad e_3 = (1, -5,7)$$
64. Write the vector $v = (4,2,1)$ as a linear combinations of the vectors
$$e_1 = (1, -3,1), \quad e_2 = (0,1,2), \quad e_3 = (5,1,37)$$
65. Determine whether the matrix, $\begin{bmatrix} -1 & 7 \\ 8 & -1 \end{bmatrix}$ is a linear combination of $\begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $\begin{bmatrix} 2 & -3 \\ 0 & 2 \end{bmatrix}$ and $\begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$ in the vector space M_{22} of 2×2 matrix
66. Write a matrix $E = \begin{bmatrix} 3 & 1 \\ 1 & -1 \end{bmatrix}$ as a linear combination of the matrices $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}$, $c = \begin{bmatrix} 0 & 2 \\ 1 & -1 \end{bmatrix}$
67. Determine whether the vector $v_1 = (1,2,3)$, $v_2 = (3,1,7)$, and $v_3 = (2,5,8)$ are linearly dependent or linearly independent.
68. Show that the set $S = \{(1,2,4), (1,0,0), (0,1,0), (0,0,1)\}$ is linearly dependent.
69. Show that the set $S = \{(1,2, -3), (1, -3,2), (2,1,5)\}$ is linearly independent.
70. Determine whether the vector $v_1 = (1,4,9)$, $v_2 = (3,1,4)$, and $v_3 = (9,3,12)$ are linearly dependent or linearly independent.
71. Determine whether the vector $v_1 = (1,1,1)$, $v_2 = (1,2,3)$, and $v_3 = (2, -1,1)$ form a basis of R^3
72. Determine whether the vector $v_1 = (1,1,2)$, $v_2 = (1,2,5)$, and $v_3 = (5,3,4)$ form a basis of R^3
73. Determine whether the vector $v_1 = (2,2,1)$, $v_2 = (1,3,7)$, and $v_3 = (1,2,2)$ form a basis of R^3

74. Let w be the subspace of R^4 generated by the vectors
$(1,-2,5,-3)$, $(2,3,1,-4)$, $(3,8,-3,-5)$ Find the basis and the dimensions of w
75. Find the basis and the dimensions of the subspace w of R^4 spanned by
$(1,4,-1,3)$, $(2,1,-3,-1)$, $(0,2,1,-5)$
76. Find the basis and the dimensions of the subspace w of R^5 spanned by
77. $v_1 = (1,2,-1,3,4)$, $v_2 = (2,4,-2,6,8)$, $v_3 = (1,3,2,2,6)$, $v_4 = (1,4,5,18)$, $v_5 = (2,7,3,3,9)$
78. Find basis and dimension for the row space and column space and null space of the following matrices.

a) $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & 6 & -3 & 5 \\ 1 & 2 & -1 & -1 \\ 5 & 10 & -5 & 7 \end{bmatrix}$ c) $\begin{bmatrix} 1 & 2 & -1 & -2 & -1 \\ 2 & 4 & -2 & -3 & -1 \\ 5 & 10 & -5 & -3 & -1 \\ -3 & -6 & 3 & 2 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & -0 \end{bmatrix}$ e) $\begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}$ f) $\begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & 3 \end{bmatrix}$

g) $\begin{bmatrix} 1 & -1 & 3 & 2 \\ 2 & -1 & 1 & 1 \\ 0 & 1 & -1 & -3 \end{bmatrix}$ h) $\begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

79. Find the coordinates of the vector $(5,-2,9)$ w.r.t basis $S = \{(2,4,-2),(1,-6,7),(1,0,2)\}$
80. Find the coordinates of the vector $(0,1,3)$ w.r.t basis $S = \{(1,1,0),(0,1,1),(1,0,1)\}$
81. Find the coordinates of the vector $(1,-3,2)$ w.r.t basis $S = \{(1,1,1),(1,1,0),(1,0,0)\}$
82. Find the inner products $\langle v_1, v_2 \rangle$, $\langle v_1, v_3 \rangle$ and $\langle v_2, v_4 \rangle$ where
$v_1 = (1,1,1,1), v_2 = (1,2,4,5), v_3 = (1,-3,-4,-2)$
83. Consider the vectors $U = (1,2,4), V = (2,-3,5), W = (4,2,-3)$ in R^3 find
I. $\langle U, V \rangle$ II. $\langle U, W \rangle$ III. $\langle V, W \rangle$ IV. $\langle U + V, W \rangle$

MODULE-5

84. Verify whether the transformation $T: R^2 \rightarrow R^2$ which is defined by $T(x, y) = (3x + 4y, 10x - 4y + 3)$ is linear or not .
85. Verify whether the transformation $T: R^2 \rightarrow R^2$ which is defined by $T(x, y) = (3x + 2y, 3x - 4y)$ is linear or not .
86. Verify whether the transformation $T: R^2 \rightarrow R^2$ which is defined by $T(x, y) = (x + 3, y - 2)$ is linear or not .
87. Check whether the transformation $T: V_1(R) \rightarrow V_3(R)$ defined by $T(x) = (x, x^2, x^3)$ Is linear or not
88. Show that the transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x + y, x - y)$ is a linear transformation or not.
89. Show that the transformation $T: V_3(R) \rightarrow V_2(R)$ defined by $T(x, y, z) = (y - x, y - z)$ is a linear transformation or not.
90. Show that the transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(a, b) = (acos\theta - bsin\theta, asin\theta + bcos\theta)$ is a linear transformation or not.
91. If T_1 and T_2 are two linear transformations from $V_3(R)$ to $V_2(R)$ defined by $T_1(x, y, z) = (2x + y, y + 2z)$ and $T_2(x, y, z) = (y + 2z, 2z + x)$, determine $T_1 + T_2$ and $3T_1 - 2T_2$
92. Find the matrix of the linear transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x, -y)$ relative to the basis :
$I)X = \{e_1, e_2\}$ and $Y = \{(1,1), (1, -1)\}, II)X = \{(1,1), (1,0)\}$ and $Y = \{(2,3), (4,5)\}$
93. Find the matrix of the linear transformation $T: V_2(R) \rightarrow V_3(R)$ defined by $T(x, y) = (-x + 2y, 3x + 3y)$ relative to the basis :
$$X = \{(1,1), (1, -1)\} \text{ and } Y = \{(1,1,1), (1, -1,1), (0,0,1)\}$$
94. Consider a matrix $A = \begin{bmatrix} 2 & 4 \\ 5 & 6 \end{bmatrix}$ which determine a linear operator on R^2 find the matrix of linear transformation relative to the basis $S = \{u_1, u_2\} = \left\{\begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ -7 \end{bmatrix}\right\}$
95. Check whether T is singular or non- singular linear transformation $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + y, y + z, z + x)$
96. Let $T: V_2(R) \rightarrow V_3(R)$ defined by $T(x, y, z) = (x + y, x - y, y)$ verify T is a singular or non-singular.
97. Let $T: R^3 \rightarrow R^3$ defined by $T(x, y, z) = (x + 2y - z, y + z, , x + y - 2z)$ verify T is a singular or non-singular.
98. Show that T is invertible and find T^{-1} let $T: R^3 \rightarrow R^3$ be defined by
$$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z)$$

99. Show that T is invertible and find T^{-1} let $T: R^3 \to R^3$ be defined by

$$T(x, y, z) = (x + z, x - z, y)$$

100. Show that $T: R^3 \to R^3$ defined by

$T(x, y, z) = (x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta, z)$ is non-singular for all the values of θ .

101. Prove that the transformation defined by $T(x, y) = (ax + by, cx + dy)$, $a, b, c, d \in R$ is non-singular transformation iff $\begin{vmatrix} a & b \\ c & d \end{vmatrix} \neq 0$

102. Find the range ,null space , rank nullity in the case of the following linear transformations . also verify the rank nullity theorem.

I) $T: V_3(R) \to V_2(R)$ *defined by* $T(x, y, z) = (y - x, y - z)$

II) $T: V_3(R) \to V_3(R)$ *defined by* $T(x, y, z) = (x + y, x - y, 2x + z)$

III) $T: V_3(R) \to V_3(R)$ *defined by* $T(x, y, z) = (x + 2y + z, z - x, y + z)$

IV) $T: V_3(R) \to V_4(R)$ *defined by* $T(e_1) = (0,1,0,2), T(e_2) = (0,1,1,0)$,

$$T(e_3) = (0,1,-1,4).$$

V) $T: V_2(R) \to V_3(R)$ *defined by* $T(1,2) = (15, -20,5)$, $T(2,1) = (12, -16,4)$