MODULE – I

1. Prove with usual notations $tan\emptyset = r\,\frac{d\theta}{dr}$.
2. Derive the expression $p = rsin\emptyset$ (Pedal Equation).
3. Prove that the following curve intersect orthogonally:
   i. $r^n = a^n cosn\theta$ and $r^n = b^n sinn\theta$
   ii. $r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$
   iii. $r = a(1 + sin\theta)$ and $r = a(1 - sin\theta)$
4. Find the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ at the point (a, 0).
5. Find the Pedal equation of the following curve
   i. $\frac{2a}{r} = 1 - sin\theta$
   ii. $\frac{2a}{r} = 1 + cos\theta$
   iii. $r^m = a^m(cosm\theta + sinm\theta)$
6. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point (3a/2,3a/2).
7. Find the radius of curvature of the cardioid $r = a(1 + cos\theta)$.
8. Derive the radius of curvature in Cartesian form.
9. Derive the radius of curvature in Polar form.
10. Find the radius of curvature of the curve $y = a\,log\,\sec(\frac{x}{a})$.
11. Show that for the equiangular spiral $r = ae^{\theta\,cot\alpha}$ where a and α are constants, $\frac{\rho}{r}$ is constant.
12. Show that ρ for the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1}.
13. Show that for the curve $r = a(1 + cos\theta)$, $\frac{\rho^2}{r}$ is a constant.
14. Find the angle between the following pairs of curves:
   i. $r = sin\theta + cos\theta$ and $r = 2sin\theta$.
   ii. $r = alog\theta$ and $r = \frac{a}{log\theta}$.
   iii. $\frac{2a}{r} = 1 - cos\theta$ at $\theta = \frac{2\pi}{3}$.

MODULE – II

1. Using Maclaurin's series prove that $\sqrt{1+sin2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \cdots$
2. Using Maclaurin's series prove that $\log(1+e^x) = log_e 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$
3. Expand e^{sinx} as Maclaurin's series upto the terms containing x^4 .
4. Expand $\log(1+sinx)$ as Maclaurin's series upto the terms containing x^4 .
5. Expand $\log(secx)$ as Maclaurin's series upto the terms containing x^6 .
6. Evaluate $(i) \lim_{x \to 0} (cosx)^{\frac{1}{x^2}}$ $(ii) \lim_{x \to \frac{\pi}{2}} (tanx)^{tan2x}$
7. Evaluate $(i) \lim_{x \to 0} [\frac{a^x+b^x c^x}{3}]^{\frac{1}{x}}$ $(ii) \lim_{x \to \frac{\pi}{2}} (secx)^{cotx}$
8. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
9. If $u = \log(x^3 + y^3 + z^3 - 3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$
10. If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$
11. If $u = xy + yz + zx$ where $x = cost, y = sint, z = t$ find $\frac{\partial u}{\partial t}$ at $t = \frac{\pi}{4}$
12. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ show that $xu_x + yu_y = 1$
13. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
14. If $u = f(x-y,\ y-z,\ z-x)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
15. if $z = f(x,y)$ where $x = rcos\theta$ and $y = rsin\theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$
16. If $u = f(\frac{y-x}{xy}, \frac{z-x}{xz})$ Prove that $x^2u_x + y^2u_y + z^2u_z = 0$
17. If $u = f(2x-3y,\ 3y-4z,\ 4z-2x)$, Show that $6u_x + 4u_y + 3u_z = 0$
18. Find the total derivative of the following function and also verify the result by direct substitution method $z = xy^2 + x^2y$ where $x = at$, $y = 2at$
19. If $u = \frac{yz}{x},\ v = \frac{zx}{y},\ w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$
20. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$
21. If $x = rsin\theta\ cos\phi$, $y = sin\theta\ sin\phi$, $z = rcos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 sin\theta$.
22. If $x + y + z = u$, $y + z = uv$ and $z = uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.
23. Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.
24. Find the maximum and minimum values of the function $x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$.
25. Examine the function $xy(a-x-y)$ for extreme values
26. Examine the function $sinx + siny + \sin(x+y)$ for extreme values

MODULE -3

1. Test the consistency and the find the solution.

I. $x + y + z = 9$; $x - 2y + 2z = 8$; $2x + y - z = 3$

II. $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

III. $x_1 + x_2 + x_3 + x_4 = 2$; $2x_1 - x_2 + 2x_3 - x_4 = -5$;

$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$; $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$

IV. $x - y + z = 2$; $3x - y + 2z = -6$; $3x + y + z = -18$

V. $x + y + z = -3$; $3x + y - 2z = -2$; $2x + 4y + 7z = 7$

2. Investigate the values of λ and μ so that following system of equation possesses

(I) Unique (II) Infinitely many solution (III) No solution

I . $2x + 3y + 5z = 9$; $7x + 3y - 2z = 8$; $2x + 3y + \lambda z = \mu$

II. $x + y + z = 6$; $x + 2y + 3z = 8$; $x + 2y + \lambda z = \mu$

III. $x + 2y + 3z = 6$; $x + 3y + 5z = 9$; $2x + 5y + \lambda z = \mu$

3. By using Gauss elimination method solve

I. $2x + 20y - 2z + 44 = 0$; $10x + 2y - z - 9 = 0$; $-2x + 3y + 10z - 22 = 0$

II. $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$

III. $2x + y + 4z = 12$; $4x + 11y - z = 33$; $8x - 3y + 3z = 20$

IV. $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

V. $3x + 4y + 5z = 3$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$

4. Find the rank of the following matrices.

a) $\begin{bmatrix} 2 & -1 & 1 \\ 3 & -1 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -1 & 3 & -1 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 1 & 0 & 2 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 5 & -4 & 6 \\ 1 & 2 & -2 & 2 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ f) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & 5 & -4 & 7 \\ -1 & -2 & -1 & 2 \\ 3 & 3 & -5 & 10 \end{bmatrix}$

g) $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ h) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ i) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$

j) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ k) $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 1 & 2 & 3 & 11 \end{bmatrix}$ l) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 4 \\ 2 & 1 & 5 \end{bmatrix}$

m) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ n) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ o) $\begin{bmatrix} 3 & 2 & 1 & 4 \\ 1 & 4 & 3 & 2 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$

p) $\begin{bmatrix} 8 & 2 & 1 & 6 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 3 \\ 5 & 1 & 1 & 4 \end{bmatrix}$ q) $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ 4 & -1 & -3 & 2 \end{bmatrix}$

ANSWERS: (48)

a) 3	b) 4	c) 4	d) 2	e) 3	f) 3
g) 3	h) 2	i) 3	j) 2	k) 3	l) 2
m) 2	n) 3	o) 2	p) 3	q) 3	

5. Find all Eigen value and corresponding Eigen vector for the following matrices

(i) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(III) $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

6. By using Gauss Seidel method solve
   I. $10x + y + z = 12$; $x + 10y + z = 12$; $x + y + 10z = 12$
   II. $27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$

III. $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$

IV. $5x + 2y + z = 12$; $x + 4y + 2z = 15$; $x + 2y + 5z = 20$

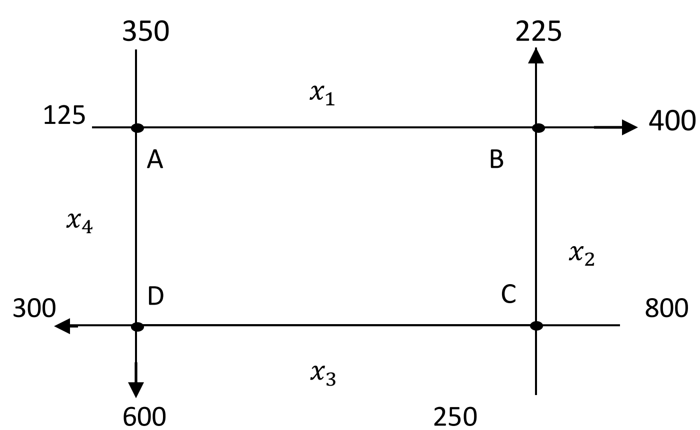
V. $28x + 4y - z = 32$; $2x + 17y + 4z = 35$; $x + 3y + 10z = 24$

7. By the Rayleigh's power method solve

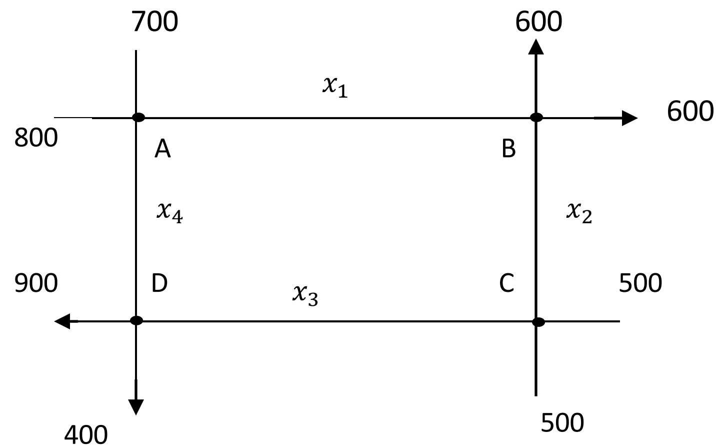
(ii) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ (iv) $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$

8. The flow of traffic through a network of streets is shown below:
   (i) Find x_1, x_2, x_3, x_4 to balance the traffic flow.
   (ii) Find the traffic flow at $x_4 = 0$
   (iii) Find the traffic flow at $x_4 = 100$

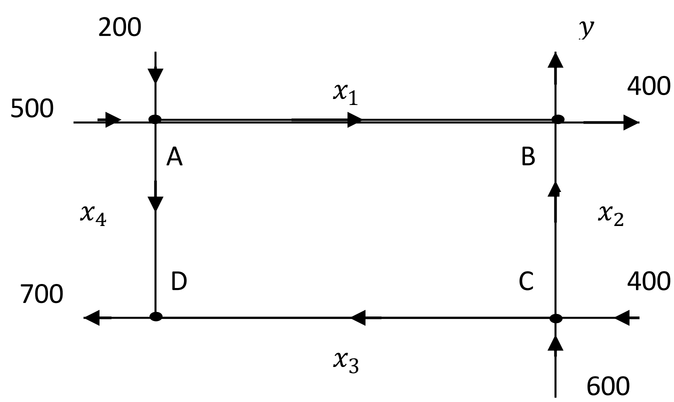


9. Balance the traffic flow at each intersection.



10. (i) What flow to the north should the traffic light at the intersection B let through to balance the traffic flow in this network?

(i) Assuming that the traffic light at B has been set to balance the total flow in and flow out, what should the flow be through the inner branches?



MODULE -4

1. Solve the following Differential Equations

(i). $y''' - 2y'' + 4y' - 8y = 0$ (ii). $y''' + 6y'' + 11y' + 6y = 0$

(iii). $\frac{d^3y}{dx^3} + y = 0$ (iv). $(4D^4 - 4D^3 - 23D^2 + 12D + 36)y = 0$

(v). $D^4 + 64)y = 0$ (vi). $4\frac{d^4y}{dx^4} - 8\frac{d^3y}{dx^3} - 7\frac{d^2y}{dx^2} + 11\frac{dy}{dx} + 6 = 0$

2. Solve :

(I). $y'' + 2y' + y = \cosh(\frac{x}{2})$ (ii). $6\frac{d^2y}{dx^2} + 17\frac{dy}{dx} + 12y = e^{-x}$

(iii). $y'' - 4y' + 13y = cos2x$ (iv). $D^2 + 3D + 2)y = 4cos^2x$

(v). $y'' + 3y' + 2y = 12x^2$

(vii). $y'' + y' + y = x^2 + x + 1$ (viii). $(D^3 + D^2 + 4D + 4)y = x^2 - 4x - 6$

(ix).$\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = sin2x$ (x). $(D^2 + 4)y = sin^2x$

3. Solve by the method of variation of parameters.

(i). $y'' + y = xsinx$ (ii). $\frac{d^2y}{dx^2} + y = tanx$

(iii). $\frac{d^2y}{dx^2} + y = secxtanx$ (iv). $\frac{d^2y}{dx^2} + 4y = sec^2 2x$

(v). $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x tanx$ (vi). $\frac{d^2y}{dx^2} + 4y = tan2x$

4. Solve the Cauchy's form of linear equation.

(I). $x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1 + x)^2$ (ii).$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2logx$

(iii). $x^2\frac{d^3y}{dx^3} + 3x\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2logx$ (iv). $x\frac{d^2y}{dx^2} - 2\frac{y}{x} = x^2 + \frac{1}{x^2}$

(v). $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(logx)$

5. Solve the Legendre's form of linear equation:

(i). $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin[\log(1+x)]$

(ii). $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=\sin2[\log(1+x)]$

(iii). $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(2x+1)^2$

(iv). $(3x+2)^2\frac{d^2y}{dx^2}+3(3x+2)\frac{dy}{dx}-36y=8x^2+4x+1$

(v).). $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos[\log(1+x)]$

MODULE -5

1. Reducible to Exact differential eqation :
   (i). Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$
   (ii). Solve $(x^2 + y^2 + xdx + xydy = 0$
   (iii). Solve $y(x + y)dx + (x + 2y - 1)dy = 0$
   (iv). Solve$(x^3 + y^3 + 6x)dx + y^2xdy = 0$
   (v). Solve $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$

2. Reducible to Bernoulli's differential eqation :
   (i). Solve $\frac{dy}{dx} + ytanx = secxy^3$
   (ii). Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$
   (iii). Solve $rsin\theta - cos\theta\frac{dr}{d\theta} = r^2$
   (iv). Solve $x\frac{dy}{dx} + y = x^3y^6$
   (v). Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$

3. (i).Find the orthogonal trajectories of the family $y^2 = cx^3$
   (ii). Find the orthogonal trajectories of the family of a curve $y^2 + 2xy - x^2 = c$
   (iii). Find the orthogonal trajectories of the family of ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$
   (iv). Find the orthogonal trajectories of the family $r^n = a^ncosn\theta$
   (v). Find the orthogonal trajectories of the family $\frac{2a}{r} = 1 - cos\theta$
   (iv). Find the orthogonal trajectories of the family $r^n = a^nsinn\theta$

4. Solvable for p :
   (i). Solve $y(\frac{dy}{dx})^2 + (x - y)\frac{dy}{dx} - x = 0$
   (ii). Solve $xy(\frac{dy}{dx})^2 - (x^2 + y^2)\frac{dy}{dx} + xy = 0$
   (iii).Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$
   (iv). Solve $(\frac{dy}{dx})^2 - 5\left(\frac{dy}{dx}\right) + 6 = 0$
   (v). Solve $x^2(\frac{dy}{dx})^2 + xy\frac{dy}{dx} - 6y^2 = 0$

5. Equations reducible to Clairaut's Equation :

(i). Solve the equation $(px - y)(py + x) = 2p$ by reducing into clairaut's form using the substitution $X = x^2, Y = y^2$.

(ii). Find the general and singular solution of the equation $x^2(y - px) = p^2y$ by reducing it into clairaut's form using substitutions $X = x^2, Y = y^2$.

(iii). Solve the equation $(px - y)(py + x) = a^2p$ by reducing into clairaut's form using the substitution $X = x^2, Y = y^2$.

(iv). Modify the following equation into clairaut's form. Hence obtain the associated general and singular solutions $xp^2 - py + kp + a = 0$.