MODULE – I

1. Prove with usual notations $tan\emptyset = r\frac{d\theta}{dr}$.
2. Derive the expression $p = rsin\emptyset$ (Pedal Equation).
3. Prove that the following curve intersect orthogonally:
   i. $r^n = a^n cosn\theta$ and $r^n = b^n sinn\theta$
   ii. $r = a(1 + cos\theta)$ and $r = b(1 - cos\theta)$
   iii. $r = a(1 + sin\theta)$ and $r = a(1 - sin\theta)$
4. Find the radius of curvature of the curve $y^2 = \frac{a^2(a-x)}{x}$ at the point (a, 0).
5. Find the Pedal equation of the following curve
   i. $\frac{2a}{r} = 1 - sin\theta$
   ii. $\frac{2a}{r} = 1 + cos\theta$
   iii. $r^m = a^m(cosm\theta + sinm\theta)$
6. Find the radius of curvature of the curve $x^3 + y^3 = 3axy$ at the point (3a/2,3a/2).
7. Find the radius of curvature of the cardioid $r = a(1 + cos\theta)$.
8. Derive the radius of curvature in Cartesian form.
9. Derive the radius of curvature in Polar form.
10. Find the radius of curvature of the curve $y = a\ log\ \sec(\frac{x}{a})$.
11. Show that for the equiangular spiral $r = ae^{\theta\ cot\alpha}$ where a and α are constants, $\frac{\rho}{r}$ is constant.
12. Show that ρ for the curve $r^n = a^n \cos n\theta$ varies inversely as r^{n-1} .
13. Show that for the curve $r = a(1 + cos\theta)$, $\frac{\rho^2}{r}$ is a constant.
14. Find the angle between the following pairs of curves:
    i. $r = sin\theta + cos\theta$ and $r = 2sin\theta$.
    ii. $r = alog\theta$ and $r = \frac{a}{log\theta}$.
    iii. $\frac{2a}{r} = 1 - cos\theta$ at $\theta = \frac{2\pi}{3}$.

MODULE – II

1. Using Maclaurin's series prove that $\sqrt{1+sin2x} = 1 + x - \frac{x^2}{2} - \frac{x^3}{6} + \frac{x^4}{24} + \cdots$
2. Using Maclaurin's series prove that $\log(1+e^x) = log_e 2 + \frac{x}{2} + \frac{x^2}{8} - \frac{x^4}{192} + \cdots$
3. Expand e^{sinx} as Maclaurin's series upto the terms containing x^4 .
4. Expand $\log(1+sinx)$ as Maclaurin's series upto the terms containing x^4 .
5. Expand $\log(secx)$ as Maclaurin's series upto the terms containing x^6 .
6. Evaluate $(i)\ \lim_{x \to 0}(cosx)^{\frac{1}{x^2}}$ $\quad (ii)\ \lim_{x \to \frac{\pi}{2}}(tanx)^{tan2x}$
7. Evaluate $(i)\ \lim_{x \to 0}[\frac{a^x+b^xc^x}{3}]^{\frac{1}{x}}$ $\quad (ii)\ \lim_{x \to \frac{\pi}{2}}(secx)^{cotx}$
8. If $u = \frac{1}{\sqrt{x^2+y^2+z^2}}$ then show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$
9. If $u = \log(x^3+y^3+z^3-3xyz)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$
10. If $u = e^{ax+by} f(ax-by)$, prove that $b\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial y} = 2abu$
11. If $u = xy + yz + zx$ where $x = cost, y = sint, z = t$ find $\frac{\partial u}{\partial t}$ at $t = \frac{\pi}{4}$
12. If $u = \log\left(\frac{x^2+y^2}{x+y}\right)$ show that $xu_x + yu_y = 1$
13. If $u = f\left(\frac{x}{y}, \frac{y}{z}, \frac{z}{x}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
14. If $u = f(x-y,\ y-z,\ z-x)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$
15. if $z = f(x,y)$ where $x = rcos\theta$ and $y = rsin\theta$ show that $\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 = \left(\frac{\partial z}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial z}{\partial \theta}\right)^2$
16. If $u = f(\frac{y-x}{xy}, \frac{z-x}{xz})$ Prove that $x^2u_x + y^2u_y + z^2u_z = 0$
17. If $u = f(2x-3y,\ 3y-4z,\ 4z-2x)$, Show that $6u_x + 4u_y + 3u_z = 0$
18. Find the total derivative of the following function and also verify the result by direct substitution method $z = xy^2 + x^2y$ where $x = at$, $y = 2at$
19. If $u = \frac{yz}{x},\ v = \frac{zx}{y},\ w = \frac{xy}{z}$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$
20. Find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$
21. If $x = rsin\theta\ cos\phi$, $y = sin\theta\ sin\phi$, $z = rcos\theta$ show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = r^2 sin\theta$.
22. If $x+y+z = u$, $y+z = uv$ and $z = uvw$, then show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$.
23. Find the extreme values of the function $f(x,y) = x^3+y^3-3x-12y+20$.
24. Find the maximum and minimum values of the function $x^3+3xy^2-15x^2-15y^2+72x$.
25. Examine the function $xy(a-x-y)$ for extreme values
26. Examine the function $sinx + siny + \sin(x+y)$ for extreme values

MODULE -4

1. Test the consistency and the find the solution.

I. $x + y + z = 9$; $x - 2y + 2z = 8$; $2x + y - z = 3$

II. $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

III. $x_1 + x_2 + x_3 + x_4 = 2$; $2x_1 - x_2 + 2x_3 - x_4 = -5$;

$3x_1 + 2x_2 + 3x_3 + 4x_4 = 7$; $x_1 - 2x_2 - 3x_3 + 2x_4 = 5$

IV. $x - y + z = 2$; $3x - y + 2z = -6$; $3x + y + z = -18$

V. $x + y + z = -3$; $3x + y - 2z = -2$; $2x + 4y + 7z = 7$

2. Investigate the values of λ and μ so that following system of equation possesses (I) Unique (II) Infinitely many solution (III) No solution

I. $2x + 3y + 5z = 9$; $7x + 3y - 2z = 8$; $2x + 3y + \lambda z = \mu$

II. $x + y + z = 6$; $x + 2y + 3z = 8$; $x + 2y + \lambda z = \mu$

III. $x + 2y + 3z = 6$; $x + 3y + 5z = 9$; $2x + 5y + \lambda z = \mu$

3. By using Gauss elimination method solve

I. $2x + 20y - 2z + 44 = 0$; $10x + 2y - z - 9 = 0$; $-2x + 3y + 10z - 22 = 0$

II. $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$

III. $2x + y + 4z = 12$; $4x + 11y - z = 33$; $8x - 3y + 3z = 20$

IV. $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$

V. $3x + 4y + 5z = 3$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$

4. Find the rank of the following matrices.

a) $\begin{bmatrix} 2 & -1 & 1 \\ 3 & -1 & 1 \\ 4 & -1 & 2 \\ -1 & 1 & -1 \end{bmatrix}$ b) $\begin{bmatrix} 4 & -1 & 3 & -1 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 1 & 0 & 2 & -2 \end{bmatrix}$ c) $\begin{bmatrix} 2 & 5 & -4 & 6 \\ 1 & 2 & -2 & 2 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ f) $\begin{bmatrix} 1 & -2 & -1 & 3 \\ 2 & 5 & -4 & 7 \\ -1 & -2 & -1 & 2 \\ 3 & 3 & -5 & 10 \end{bmatrix}$

g) $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ h) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ i) $\begin{bmatrix} 1 & -3 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 3 & 4 & 1 & -2 \end{bmatrix}$

j) $\begin{bmatrix} 1 & 3 & 4 & 5 \\ 3 & 2 & 5 & 2 \\ 2 & -1 & 1 & -3 \end{bmatrix}$ k) $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 1 & 2 & 3 & 11 \end{bmatrix}$ l) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 5 & 4 \\ 2 & 1 & 5 \end{bmatrix}$

m) $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$ n) $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ o) $\begin{bmatrix} 3 & 2 & 1 & 4 \\ 1 & 4 & 3 & 2 \\ 4 & 6 & 4 & 6 \\ 7 & 8 & 5 & 10 \end{bmatrix}$

p) $\begin{bmatrix} 8 & 2 & 1 & 6 \\ 2 & 1 & 0 & 1 \\ 3 & 0 & 1 & 3 \\ 5 & 1 & 1 & 4 \end{bmatrix}$ q) $\begin{bmatrix} 1 & 2 & 2 & 4 \\ 2 & 3 & 4 & 6 \\ 3 & 5 & 6 & 10 \\ 4 & -1 & -3 & 2 \end{bmatrix}$

ANSWERS: (48)

a) 3	b) 4	c) 4	d) 2	e) 3	f) 3
g) 3	h) 2	i) 3	j) 2	k) 3	l) 2
m) 2	n) 3	o) 2	p) 3	q) 3	

5. Find all Eigen value and corresponding Eigen vector for the following matrices

(i) $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$

(III) $\begin{bmatrix} -2 & 2 & 3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ (iv) $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

6. By using Gauss Seidel method solve

I. $10x + y + z = 12$; $x + 10y + z = 12$; $x + y + 10z = 12$

II. $27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$

III. $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$

IV. $5x + 2y + z = 12$; $x + 4y + 2z = 15$; $x + 2y + 5z = 20$

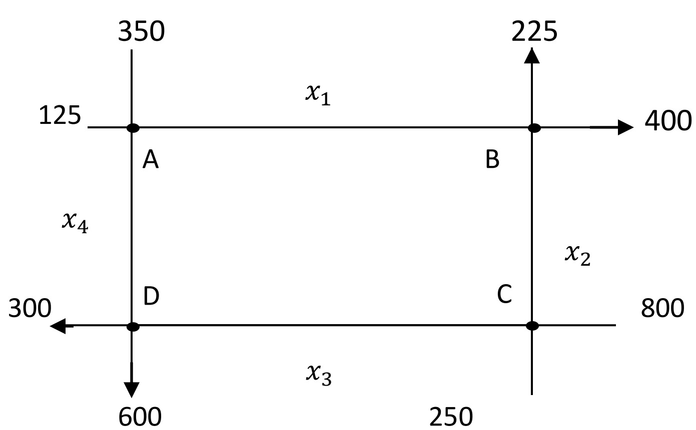
V. $28x + 4y - z = 32$; $2x + 17y + 4z = 35$; $x + 3y + 10z = 24$

7. By the Rayleigh's power method solve

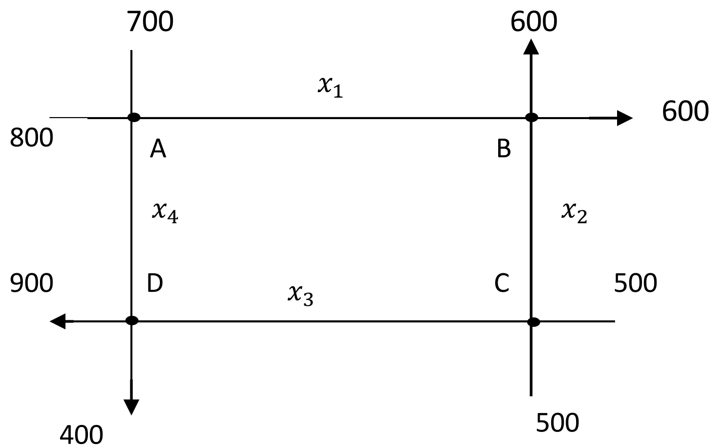
(ii) $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$ (iv) $\begin{bmatrix} 4 & 1 & -1 \\ 2 & 3 & -1 \\ -2 & 1 & 5 \end{bmatrix}$

8. The flow of traffic through a network of streets is shown below:
   (i) Find x_1, x_2, x_3, x_4 to balance the traffic flow.
   (ii) Find the traffic flow at $x_4 = 0$
   (iii) Find the traffic flow at $x_4 = 100$

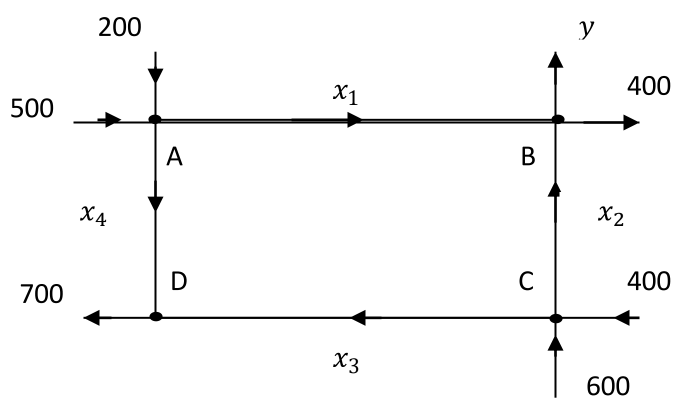


9. Balance the traffic flow at each intersection.



10. (i) What flow to the north should the traffic light at the intersection B let through to balance the traffic flow in this network?

(i) Assuming that the traffic light at B has been set to balance the total flow in and flow out, what should the flow be through the inner branches?



MODULE -4

1. By using Gauss Jordan method solve
   I. $x + y + z = 9$; $x - 2y + 3z = 8$; $2x + y - z = 3$
   II. $2x + y + 4z = 12$; $4x + 11y - z = 33$; $8x - 3y + 3z = 20$
   III. $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$
   IV. $3x + 4y + 5z = 3$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$
   V. $2x + 5y + 7z = 52$; $2x + y - z = 0$; $x + y + z = 9$
2. Diagonalize the following Matrices:
   (i) $\begin{bmatrix} -19 & 17 \\ -42 & 16 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$
3. Find the Model matrix of $\begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and verify its diagonalization
4. Find the Model matrix of $\begin{bmatrix} 7 & 0 \\ 0 & -2 \end{bmatrix}$ and verify its diagonalization
5. Find the Model matrix of $\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$ and verify its diagonalization
6. Verify Caylay – Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.
7. Verify Caylay – Hamilton theorem for the matrix A = $\begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix}$ and find its inverse.
8. Find the characteristic equation of the matrix A = of $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$ and hence find its inverse.
9. Find the characteristic equation of the matrix A = of $\begin{bmatrix} 5 & 0 \\ 0 & -1 \end{bmatrix}$ and hence find its inverse.
10. Problems on Moore – Penrose pseudoinverse :
    (i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (ii) $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ (iii) $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$

MODULE -5

1. Reducible to Exact differential eqation :
   (i). Solve $(4xy + 3y^2 - x)dx + x(x + 2y)dy = 0$
   (ii). Solve $(x^2 + y^2 + xdx + xydy = 0$
   (iii). Solve $y(x + y)dx + (x + 2y - 1)dy = 0$
   (iv). Solve$(x^3 + y^3 + 6x)dx + y^2xdy = 0$
   (v). Solve $y(2x - y + 1)dx + x(3x - 4y + 3)dy = 0$

2. Reducible to Bernoulli's differential eqation :
   (i). Solve $\frac{dy}{dx} + ytanx = secxy^3$
   (ii). Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y$
   (iii). Solve $rsin\theta - cos\theta\frac{dr}{d\theta} = r^2$
   (iv). Solve $x\frac{dy}{dx} + y = x^3y^6$
   (v). Solve $xy(1 + xy^2)\frac{dy}{dx} = 1$

3. (i).Find the orthogonal trajectories of the family $y^2 = cx^3$
   (ii). Find the orthogonal trajectories of the family of a curve $y^2 + 2xy - x^2 = c$
   (iii). Find the orthogonal trajectories of the family of ellipse $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda} = 1$
   (iv). Find the orthogonal trajectories of the family $r^n = a^ncosn\theta$
   (v). Find the orthogonal trajectories of the family $\frac{2a}{r} = 1 - cos\theta$
   (iv). Find the orthogonal trajectories of the family $r^n = a^nsinn\theta$

4. A Bacterial culture growing exponentially increases from 100 to 400 grams in 10hours . How much was present after 3hours.

5. The number N of bacteria in a culture grew at a rate proportional to N. The value of N was initially 100 and increased to 332 in one hour. What was the value of N after 90 minutes.

6. In a certain chemical reaction the rate of conversion of a substance at time 't' is proportional to the quantity of the substance still untransformed at that instant at the end of one hour 60 grams remains and at the end of 4 hours 21 grams. How many grams of the first substance was there initially.